

# The Lax pair for $C_2$ -type Ruijsenaars-Schneider model\*

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## Abstract

We study the  $C_2$  Ruijsenaars-Schneider(RS) model with interaction potential of trigonometric type. The Lax pairs for the model with and without spectral parameter are constructed. Also given are the involutive Hamiltonians for the system. Taking nonrelativistic limit, we obtain the Lax pair of  $C_2$  Calogero-Moser model.

**Keywords:** Lax pair, Ruijsenaars-Schneider(RS) model

**PACC:** 7520H; 1240E; 7510D

## I Introduction

Ruijsenaars-Schneider(RS) and Calogero-Moser(CM) models as integrable many-body models recently have attracted remarkable attention and have been extensively studied. They describe one-dimensional  $n$ -particle system with pairwise interaction. Their importance lies in various fields ranging from lattice models in statistical physics<sup>[1, 2]</sup>, to the field theory and gauge theory<sup>[3, 4]</sup>, e.g., to the Seiberg-Witten theory<sup>[5]</sup>, etc. Recently, the Lax pairs for the CM models in various root systems have been given by Olshanetsky *et al*<sup>[6]</sup>, Inozemtsev<sup>[7]</sup>, D'Hoker *et al*<sup>[8]</sup> and Bordner *et al*<sup>[9]</sup> with or without spectral parameter respectively. Further a more general algebra-geometric construction was proposed by Hurtubise *et al* in Ref. [10], while the commutative operators for the RS model based on various type Lie algebra were given by Komori<sup>[11, 12]</sup>, Diejen<sup>[13, 14]</sup> and Hasegawa *et al*<sup>[1, 15]</sup>. An interesting result is that in Ref. [16], the authors show that for the  $sl_2$  trigonometric RS and CM models there exists the same non-dynamical  $r$ -matrix structure compared with the usual dynamical ones. On the other hand, similar to Hasegawa's result that  $A_{N-1}$  RS model can be obtained as transfer matrices associated to the Sklyanin algebra, they also reveal that corresponding CM model's integrability can be depicted by  $sl_N$  Gaudin algebra<sup>[17]</sup>.

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As for the  $C_n$  type RS model, commuting difference operators acting on the space of functions on the  $C_2$  type weight space have been constructed by Hasegawa *et al* in Ref. [15]. Extending that work, the diagonalization of elliptic difference system of that type has been studied by Kikuchi in Ref. [18]. Despite of the fact that the Lax pairs for CM models have been proposed for general Lie algebra even for all of the finite reflection groups<sup>[19]</sup>, the Lax integrability of RS model are not clear except only for  $A_{N-1}$  -type<sup>[20, 2, 21, 22, 23, 24]</sup>, i.e., the Lax pairs for the RS models other than  $A_{N-1}$  -type have not yet been obtained.

In this paper, we concentrate on the  $C_2$  type trigonometric Ruijsenaars-Schneider model. The basic materials about  $C_2$  RS model are reviewed in Section II. In Section III, we present the Lax pair without spectral parameter and its integrability in Liouville sense is also given. In Section IV, taking its non-relativistic limit, we recover the system of corresponding CM type. In Section V, we give the Lax pair for the system with spectral parameter, and show that at certain limit it will degenerate to the one without spectral parameter. The last section is a brief summary and some discussions.

## II Model and equations of motion

As a relativistic-invariant generalization of the  $C_n$ -type Calogero-Moser model, the  $C_n$ -type Ruijsenaars-Schneider system is completely integrable whose integrability is shown by Ruijsenaars<sup>[25]</sup> and Diejen<sup>[13, 14]</sup>. In terms of the canonical variables  $p_i, x_i (i, j = 1, \dots, 2)$  enjoying in the canonical Poisson bracket  $\{p_i, p_j\} = \{x_i, x_j\} = 0, \{x_i, p_j\} = \delta_{ij}$ , the Hamiltonian of  $C_2$  RS system can be of the form

$$H = \sum_{i=1}^2 \{ e^{p_i} f(2x_i) \prod_{k \neq i}^2 (f(x_{ik}) f(x_i + x_k)) + e^{-p_i} g(2x_i) \prod_{k \neq i}^2 (g(x_{ik}) g(x_i + x_k)) \}, \quad (1)$$

where

$$\begin{aligned} f(x) &: = \frac{\sin(x - \gamma)}{\sin(x)}, \\ g(x) &: = f(x)|_{\gamma \rightarrow -\gamma}, \quad x_{ik} := x_i - x_k, \end{aligned}$$

and  $\gamma$  denotes the coupling constant. Notice that in Ref. [25] Ruijsenaars used another “gauge” of the momenta such that the two systems are connected by the following canonical transformation:

$$x_i \longrightarrow x_i, \quad p_i \longrightarrow p_i + \frac{1}{2} \ln \prod_{j \neq i}^2 \frac{f(x_{ij}) f(x_i + x_j) f(2x_i)}{g(x_{ij}) g(x_i + x_j) g(2x_i)}. \quad (2)$$

The canonical equations of motion for the Hamiltonian (1) are

$$\dot{x}_i = \{x_i, H\} = e^{p_i} b_i - e^{-p_i} b'_i, \quad (3)$$

$$\begin{aligned} \dot{p}_i = \{p_i, H\} = & \sum_{j \neq i}^2 \left( e^{p_j} b_j (h(x_{ji}) - h(x_j + x_i)) \right. \\ & + e^{-p_j} b'_j (\hat{h}(x_{ji}) - \hat{h}(x_j + x_i)) \\ & - e^{p_i} b_i \left( 2h(2x_i) + \sum_{j \neq i}^2 (h(x_{ij}) + h(x_i + x_j)) \right) \\ & \left. - e^{-p_i} b'_i \left( 2\hat{h}(2x_i) + \sum_{j \neq i}^2 (\hat{h}(x_{ij}) + \hat{h}(x_i + x_j)) \right) \right), \end{aligned} \quad (4)$$

where

$$\begin{aligned} h(x) : &= \frac{d \ln f(x)}{dx}, & \hat{h}(x) &:= \frac{d \ln g(x)}{dx}, \\ b_i &= f(2x_i) \prod_{k \neq i}^2 \left( f(x_i - x_k) f(x_i + x_k) \right), \\ b'_i &= g(2x_i) \prod_{k \neq i}^2 \left( g(x_i - x_k) g(x_i + x_k) \right). \end{aligned} \quad (5)$$

Here, of course  $x_i = x_i(t)$ ,  $p_i = p_i(t)$  and the dot on top denotes t-differentiation.

### III The Lax pair without spectral parameter

Let us first mention some results about the integrability of Hamiltonian (1). In Ref. [25] Ruijsenaars demonstrated that the symplectic structure of  $C_n$  type RS system can be proved integrable by embedding its phase space to a submanifold of  $A_{2n-1}$  type RS one, while in Refs. [13, 14] and Ref. [12], Diejen and Komori, respectively, gave a series of commuting difference operators which led to its quantum integrability. However, there is not any result about its Lax representation so far. That is, the explicit form of the Lax matrix  $L$ , associated with a  $M$  which ensure its Lax integrability, have not been proposed up to now. In this section, we restrict our treatment to the exhibition of the explicit form for  $C_2$  RS system. Therefore, some previous results, as well as new results, could now be obtained in a more straightforward manner by using the Lax pair.

Define one  $4 \times 4$  Lax matrix for  $C_2$  RS model as follows:

$$L = \begin{pmatrix} A & B \\ C & D \end{pmatrix}, \quad (6)$$

where  $A, B, C, D$  are  $2 \times 2$  matrices (hereafter, we use the indices  $i, j = 1, 2$ )

$$\begin{aligned}
A_{ij} &= e^{p_j} b_j \frac{\sin \gamma}{\sin(x_{ij} + \gamma)}, & B_{ij} &= e^{-p_j} b'_j \frac{\sin \gamma}{\sin(x_i + x_j + \gamma)}, \\
C_{ij} &= e^{p_j} b_j \frac{\sin \gamma}{\sin(-x_i - x_j + \gamma)}, & D_{ij} &= e^{-p_j} b'_j \frac{\sin \gamma}{\sin(x_{ji} + \gamma)}.
\end{aligned} \tag{7}$$

For the concise expression for  $M$ , we define four auxiliary  $2 \times 2$  matrices  $\tilde{A}$ ,  $\tilde{B}$ ,  $\tilde{C}$ ,  $\tilde{D}$  as follows

$$\begin{aligned}
\tilde{A}_{ij} &= e^{-p_i} b'_j \frac{-\sin \gamma}{\sin(x_{ij} - \gamma)}, & \tilde{B}_{ij} &= e^{-p_i} b_j \frac{-\sin \gamma}{\sin(x_i + x_j - \gamma)}, \\
\tilde{C}_{ij} &= e^{p_i} b'_j \frac{-\sin \gamma}{\sin(-x_i - x_j - \gamma)}, & \tilde{D}_{ij} &= e^{p_i} b_j \frac{-\sin \gamma}{\sin(x_{ji} - \gamma)},
\end{aligned} \tag{8}$$

such that  $M$  can be of the form

$$M = \begin{pmatrix} \mathcal{A} & \mathcal{B} \\ \mathcal{C} & \mathcal{D} \end{pmatrix}, \tag{9}$$

where entries of  $M$  are

$$\begin{aligned}
\mathcal{A}_{ij} &= \cot(x_{ij})(A_{ij} - \tilde{A}_{ij}), & \mathcal{D}_{ij} &= \cot(x_{ji})(D_{ij} - \tilde{D}_{ij}), & (i \neq j), \\
\mathcal{B}_{ij} &= \cot(x_i + x_j)(B_{ij} - \tilde{B}_{ij}), & \mathcal{C}_{ij} &= \cot(-x_i - x_j)(C_{ij} - \tilde{C}_{ij}), \\
\mathcal{A}_{ii} &= -\sum_{k \neq i}^2 \frac{A_{ik} - \tilde{A}_{ik}}{\sin(x_{ik})} - \sum_{k=1}^2 \frac{B_{ik} - \tilde{B}_{ik}}{\sin(x_i + x_k)}, \\
\mathcal{D}_{ii} &= \sum_{k \neq i}^2 \frac{D_{ik} - \tilde{D}_{ik}}{\sin(x_{ik})} + \sum_{k=1}^2 \frac{C_{ik} - \tilde{C}_{ik}}{\sin(x_i + x_k)}.
\end{aligned} \tag{10}$$

We have checked that  $L, M$  satisfies the Lax equation

$$\dot{L} = \{L, H\} = [M, L], \tag{11}$$

which is equivalent to the equations of motion (3) and (4) with the help of computer. The Hamiltonian  $H$  can be rewritten in the following form

$$H = \sum_{j=1}^2 (e^{p_j} b_j + e^{-p_j} b'_j) = \text{tr} L. \tag{12}$$

The characteristic polynomial of the Lax matrix  $L$  is

$$\begin{aligned}\det(L - v \cdot Id) &= \sum_{j=0}^4 (-v)^{4-j} H_j \\ &= v^4 - H v^3 + H_2 v^2 - H v + 1,\end{aligned}\tag{13}$$

where  $H_0 = H_4 = 1$ ,  $H_1 = H_3 = H$ . The function-independent Hamiltonian flows  $H$  and  $H_2$  are

$$\begin{aligned}H &= e^{p_1} f(2x_1) f(x_{12}) f(x_1 + x_2) \\ &\quad + e^{-p_1} g(2x_1) g(x_{12}) g(x_1 + x_2) \\ &\quad + e^{p_2} f(2x_2) f(x_{21}) f(x_2 + x_1) \\ &\quad + e^{-p_2} g(2x_2) g(x_{21}) g(x_2 + x_1),\end{aligned}\tag{14}$$

$$\begin{aligned}H_2 &= e^{p_1+p_2} f(2x_1) (f(x_1 + x_2))^2 f(2x_2) \\ &\quad + e^{-p_1-p_2} g(2x_1) (g(x_1 + x_2))^2 g(2x_2) \\ &\quad + e^{p_1-p_2} f(2x_1) (f(x_{12}))^2 f(-2x_2) \\ &\quad + e^{p_2-p_1} g(2x_1) (g(x_{12}))^2 g(-2x_2) \\ &\quad + 2f(x_{12}) g(x_{12}) f(x_1 + x_2) g(x_1 + x_2).\end{aligned}\tag{15}$$

We verify that  $H$  and  $H_2$  Poisson commute each other

$$\{H, H_2\} = 0,\tag{16}$$

which ensures the complete integrability of  $C_2$  RS model (in Liouville sense).

## IV Nonrelativistic limit to the Calogero-Moser system

The Nonrelativistic limit can be achieved by rescaling  $p_i \mapsto \beta p_i$ ,  $\gamma \mapsto \beta \gamma$  while letting  $\beta \mapsto 0$ , and making a canonical transformation

$$p_i \mapsto p_i + \gamma \left( \cot(2x_i) + \sum_{k \neq i}^2 (\cot(x_{ik}) + \cot(x_i + x_k)) \right),\tag{17}$$

such that

$$L \mapsto Id + \beta L_{CM} + O(\beta^2),\tag{18}$$

$$M \mapsto 2\beta M_{CM} + O(\beta^2),\tag{19}$$

and

$$H \mapsto 4 + 2\beta^2 H_{CM} + O(\beta^2).\tag{20}$$

$L_{CM}$  can be expressed as

$$L_{CM} = \begin{pmatrix} A_{CM} & B_{CM} \\ -B_{CM} & -A_{CM} \end{pmatrix}, \quad (21)$$

where

$$\begin{aligned} (A_{CM})_{ii} &= p_i, & (B_{CM})_{ij} &= \frac{\gamma}{\sin(x_i + x_j)}, \\ (A_{CM})_{ij} &= \frac{\gamma}{\sin(x_{ij})}, & (i \neq j). \end{aligned} \quad (22)$$

$M_{CM}$  is

$$M_{CM} = \begin{pmatrix} \mathcal{A}_{CM} & \mathcal{B}_{CM} \\ \mathcal{B}_{CM} & \mathcal{A}_{CM} \end{pmatrix}, \quad (23)$$

where

$$\begin{aligned} (\mathcal{A}_{CM})_{ii} &= -\sum_{k \neq i}^2 \left( \frac{\gamma}{\sin^2 x_{ik}} + \frac{\gamma}{\sin^2(x_i + x_k)} \right) - \frac{\gamma}{\sin^2(2x_i)}, & (\mathcal{B}_{CM})_{ij} &= \frac{\gamma \cos(x_i + x_j)}{\sin^2(x_i + x_j)}, \\ (\mathcal{A}_{CM})_{ij} &= \frac{\gamma \cos(x_{ij})}{\sin^2 x_{ij}}, & (i \neq j), \end{aligned} \quad (24)$$

which coincide with the form given in Ref. [6] with the difference of a constant diagonalized matrix.

The Hamiltonian of  $C_2$ -type CM model can be given by

$$\begin{aligned} H_{CM} &= \frac{1}{2} \sum_{k=1}^2 p_k^2 - \frac{\gamma^2}{2} \sum_{k \neq i}^2 \left( \frac{1}{\sin^2 x_{ik}} + \frac{1}{\sin^2(x_i + x_k)} + \frac{1}{\sin^2(2x_i)} \right) \\ &= \frac{1}{4} \text{tr} L^2. \end{aligned} \quad (25)$$

The  $L_{CM}$ ,  $M_{CM}$  satisfies the Lax equation

$$\dot{L}_{CM} = \{L_{CM}, H_{CM}\} = [M_{CM}, L_{CM}]. \quad (26)$$

## V The Lax pair with spectral parameter

Also, we can give the Lax pair which include spectral parameters. Define the Lax matrix for trigonometric RS model as follows:

$$L = \begin{pmatrix} A & B \\ C & D \end{pmatrix}, \quad (27)$$

where  $A, B, C, D$  are  $2 \times 2$  matrices ( $i, j = 1, 2$ )

$$\begin{aligned} A_{ij} &= e^{p_j} b_j \frac{\sin(x_{ij} + \gamma + \lambda) \sin \gamma}{\sin(x_{ij} + \gamma) \sin(\gamma + \lambda)}, & B_{ij} &= e^{-p_j} b'_j \frac{\sin(x_i + x_j + \gamma + \lambda) \sin \gamma}{\sin(x_i + x_j + \gamma) \sin(\gamma + \lambda)}, \\ C_{ij} &= e^{p_j} b_j \frac{\sin(-x_i - x_j + \gamma + \lambda) \sin \gamma}{\sin(-x_i - x_j + \gamma) \sin(\gamma + \lambda)}, & D_{ij} &= e^{-p_j} b'_j \frac{\sin(x_{ji} + \gamma + \lambda) \sin \gamma}{\sin(x_{ji} + \gamma) \sin(\gamma + \lambda)}. \end{aligned} \quad (28)$$

$M$  is

$$M = \begin{pmatrix} \mathcal{A} & \mathcal{B} \\ \mathcal{C} & \mathcal{D} \end{pmatrix}, \quad (29)$$

where entries of  $M$  are

$$\begin{aligned} \mathcal{A}_{ij} &= e^{p_j} b_j \frac{\sin(x_{ij} + \lambda)}{\sin \lambda \sin x_{ij}} - e^{-p_i} b'_j \frac{\sin(x_{ij} + \lambda + 4\gamma)}{\sin(\lambda + 4\gamma) \sin x_{ij}}, \\ \mathcal{D}_{ij} &= e^{-p_j} b'_j \frac{\sin(x_{ji} + \lambda)}{\sin \lambda \sin x_{ji}} - e^{p_i} b_j \frac{\sin(x_{ji} + \lambda + 4\gamma)}{\sin(\lambda + 4\gamma) \sin x_{ji}}, \quad (i \neq j), \\ \mathcal{B}_{ij} &= e^{-p_j} b'_j \frac{\sin(x_i + x_j + \lambda)}{\sin \lambda \sin(x_i + x_j)} - e^{-p_i} b_j \frac{\sin(x_i + x_j + \lambda + 4\gamma)}{\sin(\lambda + 4\gamma) \sin(x_i + x_j)}, \\ \mathcal{C}_{ij} &= e^{p_j} b_j \frac{\sin(x_i + x_j - \lambda)}{\sin \lambda \sin(x_i + x_j)} - e^{p_i} b'_j \frac{\sin(x_i + x_j - \lambda - 4\gamma)}{\sin(\lambda + 4\gamma) \sin(x_i + x_j)}, \\ \mathcal{A}_{ii} &= (\cot(\gamma) + \cot(\lambda)) e^{p_i} b_i - (\cot(\lambda + \gamma) - \cot(\gamma)) e^{-p_i} b'_i \\ &\quad + \sum_{k \neq i}^2 ((\cot(x_{ik} + \gamma) - \cot(x_{ik})) e^{p_k} b_k) \\ &\quad + \frac{\sin(x_{ik} + \lambda + 4\gamma) \sin(x_{ki} + \lambda + \gamma) \sin \gamma}{\sin(\lambda + 4\gamma) \sin x_{ik} \sin(x_{ki} + \gamma) \sin(\lambda + \gamma)} e^{-p_i} b'_k) \\ &\quad + \sum_{k=1}^2 ((\cot(x_i + x_k + \gamma) - \cot(x_i + x_k)) e^{-p_k} b'_k) \\ &\quad + \frac{\sin(x_i + x_k + \lambda + 4\gamma) \sin(x_i + x_k - \lambda - \gamma) \sin \gamma}{\sin(\lambda + 4\gamma) \sin(x_i + x_k) \sin(x_i + x_k - \gamma) \sin(\lambda + \gamma)} e^{-p_i} b_k) \\ \mathcal{D}_{ii} &= (\cot(\gamma) + \cot(\lambda)) e^{-p_i} b'_i - (\cot(\lambda + \gamma) - \cot(\gamma)) e^{p_i} b_i \\ &\quad + \sum_{k \neq i}^2 ((\cot(x_{ki} + \gamma) - \cot(x_{ki})) e^{-p_k} b'_k) \end{aligned}$$

$$\begin{aligned}
& + \frac{\sin(x_{ki} + \lambda + 4\gamma) \sin(x_{ik} + \lambda + \gamma) \sin \gamma}{\sin(\lambda + 4\gamma) \sin x_{ki} \sin(x_{ik} + \gamma) \sin(\lambda + \gamma)} e^{p_i b_k} \\
& + \sum_{k=1}^2 ((\cot(x_i + x_k) - \cot(x_i + x_k - \gamma)) e^{p_k b_k} \\
& + \frac{\sin(x_i + x_k - \lambda - 4\gamma) \sin(x_i + x_k + \lambda + \gamma) \sin \gamma}{\sin(\lambda + 4\gamma) \sin(x_i + x_k) \sin(x_i + x_k + \gamma) \sin(\lambda + \gamma)} e^{p_i b'_k}). \tag{30}
\end{aligned}$$

The  $L, M$  satisfies the Lax equation Eq.(11) and the Hamiltonian  $H$  can also be rewritten in the form of Eq.(12).

The function-independent Hamiltonian flows can be generated by calculating the characteristic polynomial of that Lax matrix  $L$

$$\det(L - v \cdot Id) = \sum_{j=0}^4 \frac{(\sin \lambda)^{(j-1)} \sin(\lambda + j\gamma)}{(\sin(\gamma + \lambda))^j} (-v)^{4-j} H_j, \tag{31}$$

where  $H_0 = H_4 = 1$ ,  $H_1 = H_3 = H$ .  $H$  and  $H_2$  have the same forms as Eq.(14) and (15).

#### Remarks:

1. As far as the forms of the Lax pair for the rational-type systems are concerned, we can get them by making the following substitutions

$$\begin{aligned}
\sin x & \rightarrow x, \\
\cos x & \rightarrow 1,
\end{aligned}$$

for all the above statements.

2. It should be pointed out that the Lax pair given in Eqs.(6)-(10) which are without spectral parameter can be derived from the one with spectral parameters (see Eqs.(27)-(30)) by taking the following limit

$$\lambda \rightarrow i\infty,$$

up to an appropriate gauge transformation of the Lax matrix with a diagonal matrix.

## VI Summary and discussions

In this paper, we propose the Lax pairs for trigonometric  $C_2$  RS model together with its rational limit and show their integrability. Involutive Hamiltonians are shown to be generated by the characteristic polynomial of the Lax matrix. In the nonrelativistic limit, the system leads to CM system associated with the root system of  $C_2$  which is known previously. It is expected that, in the general case of  $C_n$  for  $n \geq 2$ , the explicit expressions of  $L$  and  $M$  must have similar forms as those presented here. So it would be interesting to make some progress in this respect in the very near future.



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